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# Graphical determination of drying process and moisture transfer parameters for solids drying

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**Abstract**

In this paper a simple graphical method is proposed to determine the drying moisture transfer parameters such as moisture diffusivity and moisture transfer coefficient for solid products. Once the lag factor and the drying coefficient are obtained from the experimental moisture content data, the proposed graphical method can be used to estimate the drying moisture transfer parameters in a quick and efficient manner. Drying time can also be easily determined for a solid whose drying process parameters are known. Two illustrative examples are given to highlight the importance of the topic and validate the use of the present methodology for practical drying applications. 

## 1. Introduction

Although drying is commonly used in a large variety of thermal applications ranging from wood drying to food drying, food drying applications have recently received much attention.

Drying is a complex thermal process in which unsteady heat and moisture transfer occur simultaneously. Heat is transferred by convection from heated air to the product to raise the temperatures of both the solid and moisture that is present. Moisture transfer occurs as the moisture travels to the evaporative surface of the product and then into the circulating air as water vapor. The heat and moisture transfer rates are therefore related to the velocity and temperature of the circulating drying air. Moreover, the momentum transfer may take place simultaneously coupled with heat and moisture transfer. However, it is generally considered negligible for solids drying [1].

Note that the driving force for evaporation is the difference between the vapor pressure of the water in the product and the partial pressure of the water vapor in the surrounding atmosphere. Increasing the temperature of a moist solid increases the vapor pressure of the water in the solid, while increasing the temperature of air decreases the partial pressure of water vapor in the air. Moisture will migrate from an area of high vapor pressure to one of low pressure. Therefore, the moisture gradient resulting from this migration of moisture from within the heated product, where it has high vapor pressure, through micropores to the moisture surface, where it is evaporated into the surrounding atmosphere as water vapor with a lower partial pressure.

Drying solid products/objects is a very broad area and there have been many experimental and theoretical investigations appeared in the literature e.g., [1–15]. As indicated recently [2,3], determination of drying process parameters in terms of drying coefficient and lag factor; and drying moisture transfer parameters in terms of moisture diffusivity and moisture transfer coefficient of solid products subject to air drying is of great practical importance. In the literature, several studies e.g., [2–6,8–10,12–14] have been undertaken to determine/estimate drying process parameters and drying moisture transfer parameters for solids drying.

The main objective of the present paper is to develop a new graphical solution technique for determining the drying moisture transfer parameters and validate the present technique with experimental data.

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**Nomenclature**

| | |
|---|---|
| $Bi$ | Biot number |
| $D$ | moisture diffusivity ($m^2/s$) |
| $Fo$ | Fourier number |
| $G$ | lag factor |
| $h_m$ | mass transfer coefficient (m/s) |
| HT | dimensionless halving time |
| $L$ | characteristic dimension (m); half-thickness for slab (m) |
| 0LT | dimensionless lag time |
| NH | number of halving times |
| $r$ | coordinate (m) |
| RH | relative humidity (%) |
| $S$ | drying coefficient (1/s) |
| $t$ | time (s) |
| $T$ | temperature (°C) |
| $U$ | air flow velocity (m/s) |
| $W$ | moisture content by weight (kg/kg) |
| $y$ | coordinate |
| $Y$ | characteristic dimension (m) |
| $z$ | coordinate (m) |

*Greek letters*

| | |
|---|---|
| $\Phi$ | dimensionless moisture content |
| $\zeta$ | dimensionless coordinate ($= y/Y$) |

*Subscripts*

| | |
|---|---|
| a | surroundings, drying air |
| c | center |
| cal | calculated |
| e | equilibrium |
| exp | experimental |
| i | initial |

## 2. Analysis of heat and moisture transfer

### 2.1. Modeling drying process of regular shaped solid products

The governing Fickian equation for the moisture transfer in solid objects is exactly in the form of the Fourier equation of heat transfer, in which temperature and thermal diffusivity are replaced with concentration and moisture diffusivity, respectively. Therefore, similar to the case of unsteady heat transfer one can consider the most common situation for the unsteady moisture diffusion, e.g., $0 < Bi < 100$, including the finite internal and surface resistances to the moisture transfer. In this regard, the following analysis focuses on this case. The basic assumptions are (i) Thermophysical properties of the solid and the drying medium are constant. (ii) The effect of heat transfer on the moisture loss is negligible. (iii) The moisture diffusion occurs in $z$-direction (perpendicular to the slab surface) only.

The one-dimensional time-dependent moisture transfer equation in cartesian, cylindrical, and spherical coordinates for an infinite slab, infinite cylinder, and a sphere, respectively, can be written in the following compact form:

$$\frac{D}{y^m}\frac{\partial}{\partial y}\left(y^m\frac{\partial W}{\partial y}\right) = \frac{\partial W}{\partial t} \text{ and in dimensionless form}$$

$$\frac{D}{y^m}\frac{\partial}{\partial y}\left(y^m\frac{\partial \Phi}{\partial y}\right) = \frac{\partial \Phi}{\partial t}, \tag{1}$$

where $m = 0$, 1 and 2 for an infinite slab, infinite cylinder, and a sphere. $y = z$ for an infinite slab, $y = r$ for infinite cylinder and sphere. with the following initial and boundary conditions:

$$\Phi(y, 0) = 1; \quad (\partial\Phi(0, t)/\partial y) = 0;$$

and

$$-D(\partial\Phi(Y, t)/\partial y) = h_m\Phi(Y, t),$$

where $Y$ is half-thickness of slab or radius of cylinder or sphere.

Here, the dimensionless parameters can be introduced as follows:

$$\Phi = (W - W_e)/(W_i - W_e), \tag{2}$$

$$Bi = h_m L/D, \tag{3}$$

$$Fo = \frac{Dt}{L^2}. \tag{4}$$

The solution of Eq. (1) is given in the form of series solution as follows [16,17]:

$$\Phi = \sum_{n=1}^{\infty} A_n\varphi(\mu_n\zeta)\exp(-\mu_n^2 Fo), \tag{5}$$

where

$$A_n = \frac{2Bi}{\varphi(\mu_n)[\mu_n^2 + Bi^2 - (m-1)Bi]} \tag{6}$$

and the eigenfunction $\varphi(\mu_n\zeta)$ takes the form

$$\varphi(\mu_n\zeta) = \begin{cases} \cos(\mu_n\zeta) & \text{slab,} \\ J_0(\mu_n\zeta) & \text{cylinder,} \\ \frac{\sin(\mu_n\zeta)}{\mu_n\zeta} & \text{sphere,} \end{cases} \tag{7}$$

where $\zeta = y/Y$.

The eigenvalues $\mu_n$ are the solutions of the following transcendental equation

$$\frac{\partial\varphi(\mu_n\zeta)}{\partial\zeta} = -Bi\varphi(\mu_n\zeta). \tag{8}$$

Eq. (5) can be simplified by ignoring the values of the Fourier number smaller than 0.2 (taking only the first term into consideration due to the practical drying experience that the duration to reach the Fourier number of 0.2 is negligibly small compared to the rest of the time), resulting in

$$\Phi = A\varphi(\mu\zeta)\exp(-\mu^2 Fo), \tag{9}$$

where the subscripts are dropped for simplicity, i.e., $\mu$ refers to the first of the eigenvalues $\mu_n$ and $A$ in Eq. (9) is given by

$$A = \frac{2Bi}{\varphi(\mu)[\mu^2 + Bi^2 - (m-1)Bi]}. \tag{10}$$

### 2.2. Centerline moisture content

The moisture content in the centerline ($\zeta = 0$) of the object can be obtained as

$$\Phi_c = A\exp(-\mu^2 Fo). \tag{11}$$

Therefore the dimensionless time ($Fo$) for the centerline to attain a moisture content of $\Phi_c$ is

$$Fo = \frac{\ln(A/\Phi_c)}{\mu^2}. \tag{12}$$

### 2.3. Lag time

An important parameter in drying analysis is the lag time (LT) that is the time delay before the halving cycle begins, as shown in Fig. 1. In other words it is the time ($Fo$) corresponding to $\Phi_c = 1.0$ and using Eq. (12) it becomes

$$\mathrm{LT} = \frac{\ln(A)}{\mu^2}. \tag{13}$$

### 2.4. Halving time of centerline moisture content

The time needed for the centerline moisture content to decrease by 50% ($\Phi_c = 1/2$) i.e., the half-drying time is

$$Fo_{1/2} = \frac{\ln(2A)}{\mu^2}. \tag{14}$$

The half-drying time includes the initial lag time. When the lag time is subtracted from the half-drying time, the remaining corresponds to the so-called halving time. On the other hand, the consecutive halving times that follow can be expressed as $Fo_{1/4} = \ln(4A)/\mu^2$ and $Fo_{1/8} = \ln(8A)/\mu^2$ and so on. Therefore, the halving time of centerline moisture content (HT), which is another important parameter in drying applications, can be defined as

$$\mathrm{HT} = Fo_{1/2} - \mathrm{LT} = (Fo_{1/4} - Fo_{1/2}) = (Fo_{1/8} - Fo_{1/4}) = \cdots = \frac{\ln(2)}{\mu^2}. \tag{15}$$

### 2.5. Number of halving times

The drying time can also be expressed in terms of the number of halving times beyond the lag time. The number of halving times (NH) can be defined as [18]

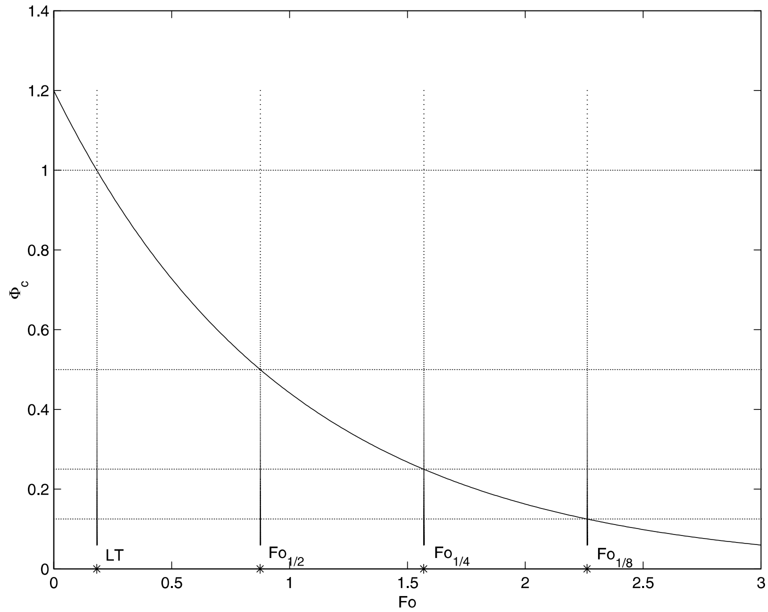


Fig. 1. A characteristic drying curve.

$$\mathrm{NH} = \frac{Fo - \mathrm{LT}}{\mathrm{HT}} = -\frac{\ln(\Phi_c)}{\ln(2)}. \quad (16)$$

Therefore the dimensionless drying time becomes

$$Fo = \mathrm{LT} + \mathrm{NH} \times \mathrm{HT}. \quad (17)$$

### 2.6. Experimental drying data correlation

Due to the fact that drying has an exponentially decreasing trend, we establish the following equation for the objects subject to drying, by introducing lag factor ($G$, dimensionless) and drying coefficient ($S$, $1/s$):

$$\Phi_c = G\exp(-St). \quad (18)$$

Here, drying coefficient shows the drying capability of the object or product and lag factor is an indication of internal resistances of object to the heat and/or moisture transfer during drying. Both Eqs. (11) and (18) are in the same form and can be equated to each other by having $G = A$ and therefore, the drying coefficient becomes

$$S = \mu^2 D/L^2. \quad (19)$$

### 2.7. Drying process parameters

Two important drying process parameters are the moisture transfer coefficient $h_m$ and moisture diffusivity $D$ both of which are related to $Bi$ number directly or indirectly. So, *moisture diffusivity* is a function of experimental drying coefficient $S$ and $\mu$ as

$$D = SL^2/\mu^2, \quad (20)$$

where $\mu$ is the first of the eigenvalues in Eq. (8).

Also, the *moisture transfer coefficient* follows the definition of $Bi$ number

$$h_m = (D/L)Bi. \quad (21)$$

## 3. Results and discussion

### 3.1. Graphical determination of drying process parameters

Based on the above analysis, Fig. 2 has been designed to estimate drying process parameters $h_m$ and $D$ for infinite slab objects. This requires that $G$ and $S$ in Eq. (18) are obtained from the experimental moisture content data. The procedure for estimating the drying process parameters from Fig. 2 is as follows:

- Starting from subplot in Fig. 2(a), LT is obtained since $G$ is equal to $A$.
- Moving through subplots in Figs. 2(b) and (c), $\mu$ and $Bi$ number are obtained, respectively.
- For the same value of $Bi$ number HT is determined from the subplot in Fig. 2(d).
- $D$ is calculated using Eq. (20).
- $h_m$ is calculated using Eq. (21).

It is important to highlight that in practice people working in the field of drying choose simple models and graphs over the complicated analytical and computational techniques to determine drying moisture parameters, although the trend today in science and engineering is towards more computational methods. In this regard, the present graphical calculation method appears to be an appropriate technique to serve such purpose, particularly for drying system design and process optimization.

Note that the drying coefficient ($S$) shows the drying capability of any object subject to drying and is different for every material and that the lag factor ($G$) shows the internal resistance to moisture diffusion within the solid. Both of these two drying process parameters are considered significant in representing and evaluating a solid-drying process.

Fig. 2 can also be used to determine the values of $G$ and $S$ for various solids subject to drying provided that the moisture diffusivity $D$ and moisture transfer coefficient $h_m$ are given. The procedure in this case is as follows: As the $Bi$ number is available, we start from the subplot in Fig. 2(d) and proceed backwards through subplots in Figs. 2(c), (b) and (a), to determine HT, $\mu$, $A$ and LT, respectively. The plots have been produced using the Eqs. (8), (10), (13), and (15) for the case of $m = 0$ (infinite slab case).

It is also important to state that the present analysis is not limited to single direction. In fact, the analysis has been carried out in a compact form applicable to cartesian, cylindrical, and spherical coordinates. Only one-dimensional diffusion has been considered. Infinite solid slab case has been treated in detail and two examples have been given. Accordingly, Fig. 2 has been provided for the case of solid slab (i.e., cartesian geometry). However, cylindrical and spherical cases can easily be treated using the present analysis (i.e., Eqs. (5)–(10)). The results can also be presented in the form of tables for more accurate calculations. Fig. 2 is provided only to demonstrate the easiness of the determination of the drying process parameters in an approximate way.

The drying time for a product to reach a centerline moisture content of $\Phi_c$ can be calculated in both of the cases outlined above using Eqs. (9) and (11) accordingly.

### 3.2. Illustrative examples

#### 3.2.1. Example 1

A carrot slab of 1 cm thickness is to be dried to a centerline moisture content of 20% of the initial moisture content. The moisture diffusivity $D$ and the moisture transfer coefficient $h_m$ are given to be $5.189 \times 10^{-9}$ $m^2/s$ and $6.6084 \times 10^{-7}$ m/s, respectively. Experimentally,

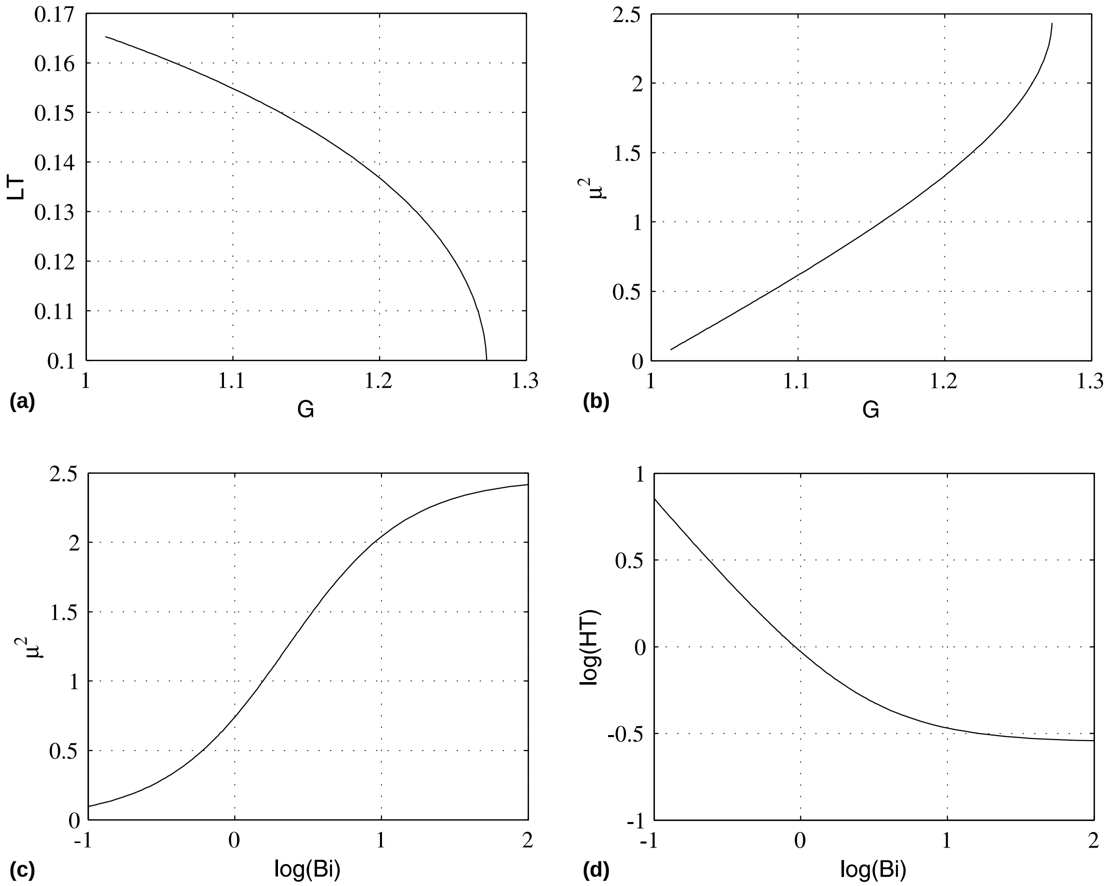


Fig. 2. Various drying parameters for a slab object. (a) Lag time versus $A$, (b) $\mu^2$ versus $A$, (c) $\mu^2$ versus $Bi$ number, and (d) halving time versus $Bi$ number.

the drying time is found to be 16200 s as obtained drying profile (e.g., moisture content change with time) as given Ruiz-Cabrera et al. [19]. Let us estimate the drying time graphically.

The log of Biot number in this case is

$$\log(Bi) = \log\left(\frac{h_m L}{D}\right) = \log\left(\frac{6.6084 \times 10^{-7} \times 0.005}{5.189 \times 10^{-9}}\right) = -0.196.$$

Referring to Fig. 2 and starting from zone (d) the following parameters are obtained as we come back through zones (c), (b), and to (a) in order:

$$\log(\mathrm{HT}) = 0.15 \quad \text{or} \quad \mathrm{HT} = 1.4125,$$

$$\mu^2 = 0.5,$$

$$A = 1.08$$

and

$$\mathrm{LT} = 0.125.$$

On the other hand,

$$\mathrm{NH} = -\frac{\ln(\Phi_c)}{\ln(2)} = -\frac{\ln(0.2)}{\ln(2)} = 2.32.$$

Therefore,

$$Fo = \mathrm{LT} + \mathrm{NH} \times \mathrm{HT} = 0.125 + 2.32 \times 1.4125 = 3.402$$

and

$$t = \frac{Fo \times L^2}{D} = \frac{3.402 \times 0.005^2}{5.189 \times 10^{-9}} = 16\,390 \text{ s}.$$

The discrepancy from the experimental value is about 1%.

The lag factor and the drying coefficient for this case are $G = A = 1.08$ and

$$S = \frac{\mu^2 D}{L^2} = \frac{0.5 \times 5.189 \times 10^{-9}}{0.005^2} = 1.038 \times 10^{-4} \text{ 1/s},$$

respectively.

*3.2.2. Example 2*

Consider a prune slab of 1 cm thickness to be dried. Experimental observation shows that the lag factor $G$ and the drying coefficient $S$ are 1.0086 and $7.91389 \times 10^{-5}$, respectively. It is also observed that the experimental drying time for a 10% centerline moisture content of the initial value is 28 800 s as obtain from Tsami and Katsioti [20]. Let us estimate the moisture diffusivity $D$ and the moisture transfer coefficient $h_m$ of this prune slab. (In the literature, $D = 3.854 \times 10^{-8}$ m$^2$/s and $h_m = 4.0261 \times 10^{-7}$ m/s, respectively.)

$A = G = 1.0086$. Starting from zone (a) in Fig. 2 and advancing trough zones in Figs. 2(b), (c), and (d), the following parameters are obtained:

$$\mathrm{LT} = 0.165,$$

$$\mu^2 = 0.05,$$

$$\log(Bi) = -1.3 \quad \text{or} \quad Bi = 0.05$$

(by extrapolation) and

$$\log(\mathrm{HT}) = 1.1 \quad \text{or} \quad \mathrm{HT} = 12.59.$$

Therefore,

$$D = \frac{S \times L^2}{\mu^2} = \frac{7.91389 \times 10^{-5} \times 0.005^2}{0.05} = 3.957 \times 10^{-8}\ \mathrm{m^2/s}$$

(2.5% discrepancy) and

$$h_m = \frac{D \times Bi}{L} = \frac{3.957 \times 10^{-8} \times 0.05}{0.005} = 3.957 \times 10^{-7}\ \mathrm{m/s}$$

(1.7% discrepancy).

On the other hand,

$$\mathrm{NH} = -\frac{\ln(\Phi_c)}{\ln(2)} = -\frac{\ln(0.1)}{\ln(2)} = 3.32.$$

Therefore the estimated drying time is

$$Fo = \mathrm{LT} + \mathrm{NH} \times \mathrm{HT} = 0.165 + 3.32 \times 12.59 = 41.96$$

or

$$t = \frac{Fo \times L^2}{D} = \frac{41.96 \times 0.005^2}{3.957 \times 10^{-8}} = 26\,512\ \mathrm{s}.$$

The discrepancy from the experimental value is found to be about 8%.

Furthermore, a comparison of the predictions of the present method with the experimental data as given in the above two illustrative examples is summarized in Table 1.

Consequently, in the present work drying process parameters such as drying coefficient and lag factor, and drying moisture transfer parameters such as moisture diffusivity and moisture transfer coefficient are determined graphically. For that four plots are proposed in the ranges $-1 < \log(Bi) < 2$, referring to the Biot number ranging from 0 to 100 as the most practical and

Table 1
Model predictions and experimental data of drying times for Carrot and prune products discussed in the illustrative examples, along with experimental conditions

| Product | Shape | $T_a$ (°C) | RH (%) | $U$ (m/s) | $L$ (m) | $D = (SL^2)/\mu$ (m$^2$/s) | $h_m = (Bi\,D)/L$ (m/s) | $\Phi_c$ | $Bi$ | LT | HT | NH | $t_{cal}$ (s) | $t_{exp}$ (s) | Error (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Carrot [19] | Slab | 50 | – | 2.5 | 0.005 | 5.189E−09 | 6.6084E−07 | 0.2 | 0.637 | 0.125 | 1.4125 | 2. 32 | 16 390 | 16 200 | 1.17 |
| Prune [20] | Slab | 60 | 15 | 3–5 | 0.005 | 3.854E−08 | 4.0261E−07 | 0.1 | 0.05 | 0.165 | 12.59 | 3. 32 | 26 512 | 28 800 | −7.94 |

most common case, and hence $1 < A < 1.3$ accordingly to be used successively for the specified lag factor and drying coefficient determined experimentally.

## 4. Conclusions

A graphical method has been developed to determine drying process parameters. Using the proposed method determination of the drying process parameters do not require solution of transcendental equations nor using an iterative solution techniques. The proposed graphical method is a handy tool in practice, especially where computational facilities are not available. The method can be used for two purposes; (a) determination of drying moisture transfer parameters using the experimental lag factor and drying coefficient data, and (b) estimating the drying time when the drying process parameters are known.

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